SimdMinimizers: Computing Random Minimizers, fast

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— Abstract

Motivation. Because of the rapidly-growing amount of sequencing data, computing *sketches* of large textual datasets has become an essential preprocessing task. These sketches are typically much smaller than the input sequences, but preserve sufficient information for downstream analysis. Minimizers are an especially popular sketching technique and used in a wide variety of applications. They sample at least one out of every w consecutive k-mers. As DNA sequencers are getting more accurate, some applications can afford to use a larger w and hence sparser and smaller sketches. And as sketches get smaller, their analysis becomes faster, so the time spent sketching the full-sized input becomes more of a bottleneck.

Methods. Our library simd-minimizers implements a random minimizer algorithm using SIMD instructions. It supports both AVX2 and NEON architectures. Its main novelty is two-fold. First, it splits the input into 8 chunks that are streamed over in parallel through all steps of the algorithm. This is enabled by using the completely deterministic *two-stacks* sliding window minimum algorithm, which seems not to have been used before for finding minimizers.

Results. Our library is up to $6.8 \times$ faster than a scalar implementation of the *rescan* method when w=5 is small, and $3.4 \times$ faster for larger w=19. Computing *canonical* minimizers is less than 50% slower than computing forward minimizers, and over $15 \times$ faster than the existing implementation in the minimizer-iter crate. Our library finds all (canonical) minimizers of a 3.2 Gbp human genome in 5.2 (resp. 6.7) seconds.

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 $\textbf{Supplementary Material } \textit{Software} : \texttt{https://github.com/rust-seq/simd-minimizers} \ [9]$

archived at swh:1:dir:d2fbef4c7750f02bc5cb084d0873078fef7f4e22

Software: https://github.com/rust-seq/packed-seq [8]

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1 Introduction

Minimizers were simultaneously introduced by [31] and [30] as a method to sample short strings of fixed length k, called k-mers or k-grams, for the purpose of fingerprinting and comparing large textual documents such as genomic sequences. This sampling method plays a central role in bioinformatics for the high-throughput analysis of DNA sequencing data and is a fundamental building block for many related tasks such as indexing [29, 21], counting [3, 23], aligning [19, 14], or assembling [4, 1] genomic sequences.

Minimizers are defined as follows: given a window W of w consecutive k-mers, the minimizer of W is the smallest k-mer according to some order. In practice, this order is often pseudo-random by hashing the k-mers, leading to the random minimizer. The density of a minimizer scheme is the expected fraction of sampled k-mers on a sufficiently long random string. When k is not too small, random minimizers have an expected density close to 2/(w+1), which is around twice as much as the lower bound of 1/w. In recent years, there have been a number of papers on methods with lower density than random minimizers [35, 28, 11, 7, 6]. While these contributions have narrowed the gap to an optimal-density sampling scheme [17], none of them focused on improving the computation time of minimizers.

In bioinformatics applications, the k-mer length is usually at most $k \leq 31$, so that each k-mer can be represented by a single u64 machine word, and the window length can be as low as 5 (where around a third of the k-mers is sampled), but is typically between 10 and 30. Longer windows of size up to 100 are also possible to index highly conserved texts.

Problem statement. We aim to solve the following problem as fast as possible: given a bitpacked representation of a sequence of ACGT DNA characters, compute the positions of all (canonical) random minimizers.

Contributions. This work introduces a carefully optimized algorithm to compute the minimizers of a genomic sequence. Conceptually, it consists of two parts. First, we introduce a scalar algorithm (Section 3) that uses O(n) time and O(w) space. Most of this can be trivially parallelized to L=8 independent lanes with AVX2 or NEON SIMD instructions, and in Section 4 we specifically handle the input and output. The parts we discuss are:

- 1. We apply ntHash [25, 16], a pseudo-random rolling hash function for k-mers (Section 3.2).
- 2. We compute the sliding-window minima of the hashes using the *two-stacks* method [12, 32] (Section 3.3).
- 3. We compute canonical minimizers based on refined minimizers [27] to decide the strand of each window (Section 3.4).
- 4. We extend the scalar algorithm to SIMD by using it on L chunks of the (bitpacked) input sequence in parallel (Section 4.1).
- **5.** Lastly, we collect and deduplicate the L parallel streams into unique minimizer positions (Section 4.2).

Results. Our method is $3.4\times$ (for large w=19) to $6.8\times$ (for small w=5) times faster than the fastest non-SIMD algorithm for computing forward minimizers. For canonical minimizers, we only compare against a simple implementation and find over $15\times$ speedup. As a result, we can compute the minimizers of a human genome in 5.2 seconds, and the canonical minimizers in 6.7 seconds. We also adapt our method to support generic plain-text ASCII input ($|\Sigma|=256$), which is slightly (30%) slower due to the larger input characters.

Software. A Rust implementation of our method is publicly available at https://github.com/rust-seq/simd-minimizers. The packed sequence representation, the splitting into chunks, and the parallel iteration over their bases is extracted to a separate library, packed-seq, available at https://github.com/rust-seq/packed-seq.

2 Preliminaries

Bitpacking. In this work, we assume that the input sequence is over the DNA alphabet $\Sigma = \{A, C, T, G\}$ and that each letter is encoded using two bits: A = 00, C = 01, T = 10, G = 11. This encoding can easily be obtained from the ASCII representation by applying a mask: (c >> 1) & 3. Additionally, we assume that the whole sequence is bitpacked using this 2-bit encoding, which can be done as a preprocessing step on the input if necessary. Non-ACGT characters have to be handled during this preprocessing as well, and could be skipped, converted to A, or the input sequence could be split at these points. We assume that the hardware is little-endian, and that the integer value of a sequence $x_0x_1x_2...x_{k-1}$ is given by $\sum_{i=0}^{k-1} x_i \cdot 4^i$.

Minimizers. Given parameters w and k, a window W of length $\ell = w + k - 1$ contains w consecutive k-mers. The minimizer of the window is the smallest k-mer in the window. For random minimizers, k-mers are ordered by a pseudo-random order, usually given by comparing hashes of the k-mers. In case of ties, the leftmost smallest k-mer is chosen.

Our goal is to compute the absolute position of the minimizer of every window W in the input text. Since adjacent windows often have the same k-mer as minimizer, we only want each position to be listed once in the output.

Canonical minimizers. Because DNA is double-stranded and most sequencing technologies do not distinguish these two strands, genomic sequences have an additional constraint: a sequence and its reverse-complement (the reversed sequence of complementary bases $A \leftrightarrow T$ and $C \leftrightarrow G$) should be considered identical. To satisfy this constraint, canonical minimizers should return the same set of k-mers regardless of the strandedness of the input. Specifically, if the canonical minimizer of a window W is at position p, then the canonical minimizer of the reverse-complement \overline{W}^r of W should be at position |W| - k - p = w - 1 - p. In practice, canonical minimizers are often overlooked as an implementation detail and most existing methods simply compare canonical k-mers, computed as $x^c = \min(x, \overline{x}^r)$, which gives a weaker guarantee [24].

3 A predictable scalar algorithm

Overview. Computing the minimizers of a sequence generally involves a few steps, as shown in Figure 1. First, the k-mers are hashed. We do this using ntHash, a rolling hash (Section 3.2). Next, in each window of w k-mer hashes, we must find the position of the leftmost k-mer with the smallest hash. For this, we use a method based on the two-stacks algorithm (Section 3.3). Lastly, many adjacent windows will have the same minimizer, and thus these positions must be deduplicated.

In this section, we present a scalar algorithm (that will be trivial to parallelize later by using SIMD instructions Section 4), and we also introduce a variant to compute canonical minimizers in Section 3.4.

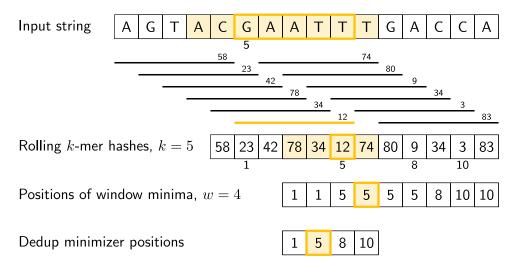


Figure 1 Overview of computing minimizers, for k=5 and w=4. First, all k-mers of the input string (shown as black horizontal lines) are hashed (the small numbers above them). Then, for each window of w=4 consecutive k-mers, we find the absolute position of its smallest k-mer. For example, for the first window, the k-mer at position 1 (zero-based) has the smallest hash (23). Similarly, the fourth window (highlighted, spanning $\ell=w+k-1=8$ bases) has minimal hash 12 by the k-mer at position 5. Lastly, these minimizer positions are deduplicated.

The rolling hash takes constant time per character, and our sliding window algorithm will also take amortized constant time per character, so that the entire method runs in O(n). It needs O(w) space to store the hashes of the current window. With the SIMD optimizations, this improves to O(n/L) time and O(Lw) space, where L is the number of SIMD lanes.

3.1 Iterating Packed Input

As input to our algorithm, we use a 2-bit packed sequence representation. Our representation is little-endian, in that the two least significant bits of each 8-bit or 32-bit value correspond to the leftmost encoded base.

Extracting bases. To iterate the bases of the input string, we process them one u32 of 16 bases at a time. For each of those, we do 16 iterations where we extract the 2 least significant bits to return, and then shift the remainder down by 2 bits.

For 8-bit ASCII input, instead, the low 8 bits can be extracted and then shifted away.

Delayed iterator. In order to support a rolling hash, we simply iterate the characters twice in parallel, with an offset: once for the character *entering* each k-mer and once delayed by k-1 iterations for the character *leaving* each k-mer. Similarly, for canonical minimizers we will also need the character leaving the window, for which we do a third iteration delayed by $\ell-1$ steps. To avoid additional reads from memory, we store u32 values read from memory in a small ring buffer.

3.2 Rolling Hash

The first step of the algorithm is the computation of a hash for each k-mer. We adapt ntHash [25, 16], a popular rolling hash function for DNA sequences that is based on cyclic polynomials [2] as an improvement over Karp-Rabin hashing [15]. This rolling hash

Algorithm 1 Pseudocode for iterating the packed input sequence, and for iterating over the last (in) and first (out) bases of all k-mers. The code samples are written in terms of "iterator adapters", where the input is a stream over something, and the output is a transformed stream of yielded elements.

```
1: function ITERBP(seq)
                                                 ▶ The bases in seg are bitpacked (4 bases in each byte).
        for block: u32 in seq do ▷ The text is split into 32-bit blocks. Padding at end not shown.
 2:
            for i in \{0, ..., 15\} do
 3:
                yield block & 3
 4:
                \texttt{block} \leftarrow \texttt{block} \ \texttt{> 2}
 5:
 6: function ITERBPDELAYED(seq, k)
                                                        \triangleright Return two streams, one delayed by k-1 steps.
 7:
        in_bps \leftarrow ITERBP(seq)
 8:
        \mathtt{out\_bps} \leftarrow \mathtt{ITERBP}(\mathtt{seq})
 9:
        for in_bp in in_bps[0:k-1] do
                                                                           \triangleright [i : j] notation is right-exclusive.
10:
           yield (in_bp,_)
                                                              \triangleright The first k-1 iterations, there is out base.
        for (in_bp, out_bp) in zip(in_bps[k-1:], out_bps) do
11:
12:
            yield (in_bp, out_bp)
```

Algorithm 2 NtHash rolling hash. The input is an iterator over pairs of characters coming in and out of each k-mer as returned by ITERBPDELAYED (Algorithm 1). The output is an iterator over the hashes of all k-mers. The algorithm first processes the first k-1 characters to initialize the rolling hash, and then repeatedly adds and removes one character at a time. Each operation can be vectorized to work on L lanes in parallel.

```
1: function NTHASH(k, in_out)
                                                                   ▷ in_out iterates pairs (last bp, first bp) of each k-mer.
 2:
           T_{\text{in}} \leftarrow [f(\mathsf{A}), f(\mathsf{C}), f(\mathsf{T}), f(\mathsf{G})]
                                                                                                                          \triangleright Static lookup tables.
           T_{\mathrm{out}} \leftarrow [\mathrm{rot}^{k-1}(f(\mathsf{A})), \mathrm{rot}^{k-1}(f(\mathsf{C})), \mathrm{rot}^{k-1}(f(\mathsf{T})), \mathrm{rot}^{k-1}(f(\mathsf{G}))]
 3:
                                                                              \triangleright 32-bit rolling hash value of (k-1) by overlaps.
 4:
           for (in_bp, \_) in in_out[0:k-1] do
                                                                                \triangleright The first k-1 iterations, there is no out_bp.
 5:
                 h \leftarrow \operatorname{rot}^1(h)
                                                                            \triangleright In SIMD, implemented as h \leftarrow (h \ll 1 \mid h \gg 31).
 6:
 7:
                h \leftarrow h \oplus \mathtt{table\_lookup}(T_{\mathrm{in}}, \mathtt{in\_bp}) \triangleright \mathit{In SIMD}, implemented with permute instructions.
           for (in_bp, out_bp) in in_out[k-1:] do
                                                                                  \triangleright 2-bit bp coming in and out of each k-mer.
 8:
 9:
                 h \leftarrow \operatorname{rot}^1(h)
10:
                 h \leftarrow h \oplus \texttt{table\_lookup}(T_{in}, \texttt{in\_bp})
11:
                 yield h
12:
                 h \leftarrow h \oplus \texttt{table\_lookup}(T_{\texttt{out}}, \texttt{out\_bp})
```

function can be summarized as follows: each of the 4 bases is associated to a fixed 32-bit random value, denoted f(x), and the hash of a k-mer $u = x_0 \dots x_{k-1}$ is computed as $h(u) = \bigoplus_{i=0}^{k-1} \operatorname{rot}^{k-1-i}(f(x_i))$, where rot^i denotes a cyclic rotation by i bits to the left and \oplus denotes xor. In practice, each x_i is a value in $\{0, 1, 2, 3\}$ and $f(x_i)$ is a simple table lookup, as shown in Algorithm 2.

Using a rolling hash has a twofold advantage in our use case: first, we only need the first and last bases of each k-mer to update its hash based on the previous one, so that we do not have to store the k-mer itself, and second, we are not limited to k-mers that fit in a constant number of words.

Most of the pseudocode in Algorithm 2 can be trivially adapted to work on u32x8 SIMD registers containing 8 32-bit values. The table_lookup function computes the values of f(x) of the L nucleotides stored in in_bp or out_bp by looking up their value in table_in or table_out. It can be implemented using _mm256_permutevar_ps in AVX2 or vqtbl1q_u8 in NEON.

Figure 2 An example showing how sliding-window minima can be computed for windows of size w = 4. The input k-mer hashes are split into blocks of size w, and for each block, prefix and suffix minima are computed. Then, each window that overlaps two blocks can be split into a suffix and prefix, and the two corresponding minima (highlighted, 5 and 1) can be looked up. The minimum of these two is the minimum of the window. Windows that coincide with a block (as shown on the right) simply use the entire block itself as both prefix and suffix. In practice, we track the position of each minimum alongside its value.

MulHash. A drawback of ntHash is that the efficient SIMD table lookup only works because it uses an alphabet of size 4. This means it does not work for general ASCII input. We introduce an alternative that we call mulHash, which replaces the table lookup of ntHash by $f(x) = C \cdot x$, where C is a fixed random constant and the multiplication is wrapping over 32-bit integers. Although slightly slower, this can be easily computed for any input value x.

3.3 Sliding Window Minimum

The second step of our algorithm computes the position of the minimum k-mer hash in each sliding window of w hashes. A number of different approaches can be used for this, and pseudocode for each of the methods discussed can be found in Algorithm 3.

Naive. The simplest approach is to simply loop over the w values in each window independently. This takes O(wn) time, but can still be quite efficient when w is small by using vectorized instructions.

Monotone queue. An approach with better complexity is to use a monotone queue, which stores a non-decreasing subsequence of the w hashes, alongside their positions. Every time the window slides one to the right and we are about to push a new k-mer hash onto the right of the queue, we first remove any values larger than it, as they are "shadowed" by the new hash and can never be minimal anymore. The minimum of the window is then always the leftmost queue element. This data structure guarantees an amortized constant time update, but has many unpredictable branches due to removing between 0 and w values, which makes it costly in practice.

Rescan. Another approach used in bioinformatics is to only keep track of the minimum value and rescan the entire window of w values when the current minimum goes out of scope [19, 20]. While this algorithm does not guarantee a worst-case constant time update, it only branches when the minimum goes out of scope and hence is more predictable. This makes it more efficient in practice, especially since minimizers typically have a density of O(1/w) so that the O(w) rescan step takes amortized constant time per element.

To the best of our knowledge, most existing methods in bioinformatics use either a monotone queue or a rescan approach [13, 19, 20].

Algorithm 3 Scalar implementations of the naive, queue and rescan sliding minimum methods, that return the position of the leftmost minimum value in each window of size w.

```
1: function NAIVE(w, vals)
 2:
        for i in \{0,\ldots,|\text{vals}|-w\} do
 3:
           yield arg min{vals}[i], \dots, vals}[i+w-1]
 4: function QUEUE(w, vals)
 5:
        queue \leftarrow empty DoubleEndedQueue
        for i \text{ in } \{0, ..., |vals| - 1\} \text{ do}
 6:
            if queue.front().pos + w \le i then
 7:
 8:
                queue.pop_front()
 9:
            while queue.back.val > vals[i] do
10:
             queue.pop_back()
11:
            \texttt{queue.push\_back}(\texttt{val} \leftarrow \texttt{vals}[i], \texttt{pos} \leftarrow i)
12:
            if i \ge w - 1 then
13:
            yield queue.front().pos
14: function RESCAN(w, vals)
        \min_{val} \leftarrow +\infty
15:
        min_pos \leftarrow 0
16:
        for i in \{0,\ldots,|\mathtt{vals}|-1\} do
17:
            if vals[i] < min_val then
18:
19:
                min_val \leftarrow vals[i]
20:
                min_pos \leftarrow i
21:
            if min_pos + w \le i then
22:
                \min_{pos} \leftarrow \arg\min{\{vals[i-w+1], \ldots, vals[i]\}}
23:
                min_val \leftarrow vals[min_pos]
24:
            if i \geq w - 1 then
            _ yield min_pos
25:
```

Two-stacks. Since our goal here is to compute L minima at the same time using vectorized instructions, we want to avoid any kind of data-dependent branches to ensure that the code path is the same for each chunk. A method for *online* sliding minima where elements may be added and removed at varying rates is the two-stacks method [12, 32], that is well-known in the competitive programming community¹. Here, we only discuss a version where the number of elements remains constant at w.

Conceptually, we first split the sequence of input k-mer hashes into blocks of size w, as shown in Figure 2. Then, we can compute both prefix minima and suffix minima of each block in O(w) per block, or O(1) amortized per input hash. Now, any window of size w can be split into a suffix of the previous block and a prefix of the current block (as highlighted in Figure 2), and we can return the minimum of the two corresponding suffix/prefix minima. When the window exactly coincides with a block (as shown on the right), the suffix and prefix minimum are equal.

In the implementation, Algorithm 4, the prefix minima are simply computed incrementally, while the suffix-minima are computed in batches after every block of w hashes has been filled. This way, only a single buffer of size w is needed, and the two cases above are unified.

The only branch in the algorithm triggers exactly every w iterations, and is thus completely data-independent. Thus, branches are both highly predictable, and the same across all L lanes.

https://codeforces.com/blog/entry/71687

Algorithm 4 Sliding minimum computation using a method based on the *two-stacks* algorithm. The input is an iterator over 32-bit hash values of the k-mers in each of the chunks as returned by NTHASH (Algorithm 2), and the output is an iterator over the position of the leftmost minimum hash in each window of w hashes. Each operation can be vectorized to work on L lanes in parallel, each identified by a LANE_IDX $\in [0, L-1]$.

```
1: function SLIDINGMIN(w, hash_it)
                                                             ▷ hash_it iterates over 32-bit hashes of each k-mer.
                                                                                        ▷ Use high 16 bits of each value.
          \mathtt{val\_mask} \leftarrow \mathtt{0xffff0000}
 2:
          {\tt pos\_mask} \leftarrow {\tt 0x0000ffff}
 3:
                                                                                            ▶ Low 16 bits store positions.

ightharpoonup Rolling prefix min.
 4:
          \texttt{prefix\_min} \leftarrow \texttt{Oxfffffff}
          \texttt{suffix\_min} \leftarrow \texttt{Buffer} \ \text{of size} \ w \ \text{filled with} \ \texttt{Oxffffffff}
 5:
 6:
          n \leftarrow |\mathtt{hash\_it}| - w - 1
                                                              \triangleright The first w-1 hashes do not complete a window.
 7:
          i \leftarrow \texttt{LANE\_IDX} \times n
                                                                                           ▷ Current position of the lane.
 8:
          j \leftarrow 0
                                                               \triangleright j := (i - \mathtt{LANE\_IDX} \times n) \mod w, index in buffer.
 9:
          \mathbf{for}\ h\ \mathrm{in}\ \mathtt{hash\_it}\ \mathbf{do}
10:
              val \leftarrow (h \& val_mask) \mid i
              prefix_min \leftarrow min(prefix_min, val)
11:
12:
              suffix_min[j] \leftarrow val
                                                                             > Write the current value into the buffer.
              j \leftarrow j + 1
13:
              if j = w then
                                                               ▶ When the buffer is full, recompute suffix minima.
14:
                   j \leftarrow 0
15:
                   {\tt prefix\_min} \leftarrow {\tt Oxfffffff}
                                                                                                          ▷ Reset prefix min.
16:
                   for k in \{w-2, w-3, \ldots, 0\} do
17:
                   | suffix_min[k] \leftarrow min(suffix_min[k], suffix_min[k+1])
18:
19:
              if i \geq \texttt{LANE\_IDX} \times n + w - 1 then
                                                                          \triangleright Skip the first w-1 incomplete windows.
20:
               yield min(prefix_min, suffix_min[j]) & pos_mask
                                                                                                      \triangleright Position of the min.
21:
```

16-bit hashes. To easily return the leftmost position of the minimum, instead of the minimum itself, we only use the upper 16 bits of each hash value, and store the position in the lower 16 bits. After taking the minimum, we mask out the high bits to obtain its position. Strings longer than 2^{16} characters can be processed in chunks of 2^{16} . We note here that while using a 16-bit hash is usually not sufficient for k-mer indexing purposes, this is sufficiently good for selecting the minimum of a window when $w \ll 2^{16}$, which is indeed the case in bioinformatics applications, where usually $w \leq 100$. Our library intentionally does not expose this hash to the user. Instead, a second, larger, hash can be computed afterwards for indexing purposes if needed.

3.4 Canonical Minimizers

One problem that arises when using minimizers in practice is that the DNA strand is often unknown. Thus, we do not know whether we are reading the forward (sense) or reverse-complement (antisense) strand, where the sequence is reversed and complementary bases $A \leftrightarrow T$ and $C \leftrightarrow G$ are used. When given a sequence, we would like to select the same minimizers in a strand-agnostic manner.

Canonical strand. Following [27], we define the *canonical* strand for each window of length ℓ as the strand where the count of GT bases (encoded values 3 and 2) is the highest, as shown in Algorithm 5. (In fact, any pair of bases can be chosen, as long as they are not complementary.) This count is in $[0,\ell]$ and when ℓ is odd there can be no tie between the two strands. In code, we instead compute the more symmetric $\#\text{GT} - \#\text{AC} = 2\#\text{GT} - \ell$, which is in $[-\ell,\ell]$, so that count > 0 defines the canonical strand. Then, we select the leftmost forward minimizer when the input window is canonical, and the rightmost reverse-complement minimizer when the input window is not canonical.

Algorithm 5 Counting #GT – #AC. When ℓ is odd, this can never be 0, and a window is canonical when the value is positive. The input is a stream over bases entering and leaving each window, where one is delayed by $\ell - 1 = w + k - 2$ steps. Each operation can be vectorized to work on L lanes in parallel.

```
1: function CANONICAL(k, w, in\_out) > in\_out iterates pairs (last bp, first bp) of each window.
       \ell \leftarrow k - w + 1
                                                ▶ Window length, it must be odd to guarantee canonicity.
2:
       c \leftarrow -\ell > Count #GT - #AC, starting at -\ell and adding/subtracting 2 for each G or T.
3:
       for (in\_bp,\_) in in\_out[0:\ell-1] do \triangleright The first \ell-1 iterations, there is no out\_bp.
4:
5:
         c \leftarrow c + (\texttt{in\_bp \& 2})
       for (in_bp,out_bp) in in_out [\ell-1:] do \Rightarrow 2-bit bp coming in and out of each window.
6:
7:
           c \leftarrow c + (\text{in\_bp \& 2})
                                                                    \triangleright A window is canonical if \#\mathsf{GT} > \#\mathsf{AC}.
8:
           yield c > 0
9:
           c \leftarrow c - (\texttt{out\_bp \& 2})
```

The benefit of this method over, say, determining the strand via the middle character (assuming again that ℓ is odd), is that the GT count is more stable across consecutive windows, since it varies by ± 1 . This way, the strandedness and thus the chosen minimizer is less likely to flip between adjacent windows.

Canonical ntHash. NtHash can be easily modified to compute both the forward and reverse-complement hash of each k-mer at the same time. Then, we can duplicate the sliding minimum algorithm to find the minimum of $(h_{\text{fwd}}(kmer[i]), i)$ and the maximum of $(-h_{\text{rc}}(kmer[i]), i)$ over each window. This way, ties in the forward direction are broken towards small i and ties in the reverse-complement direction are broken towards large i. To avoid selecting a new minimizer whenever the strand changes, we use a strand-independent canonical version of ntHash for both strands, defined as $h_{\text{c}} = h_{\text{fwd}} + h_{\text{rc}}$ [16], and we do not distinguish which strand a minimizer k-mer was chosen from.

While this process does not assume k to be odd, this may be a useful additional assumption for further processing of the minimizers, so that the canonical representation of each k-mer itself can be indexed. Alternatively, k-mers could all be processed in the direction of the strand of the window they minimize, but this requires additional bookkeeping to store this direction, and would require duplicating k-mers when they minimize both forward and reverse-complement strands.

Canonical minimizers are not forward. One small drawback of this scheme is that it is not forward. Suppose a long window has many occurrences of the smallest minimizer, and that shifting the window one position changes its canonical strand. Then, the position of the sampled minimizer could jump backwards: from sampling the rightmost minimizer to sampling the leftmost minimizer. In practice, this does not seem to be a major limitation, both because it is rare and because downstream methods usually work fine on non-forward schemes anyway.

4 A SIMD algorithm

SIMD. So far, our algorithms for iterating the input bases (Algorithm 1), hashing k-mers (Algorithm 2), and computing sliding window minima (Algorithm 4) are completely scalar. Now, we would like to use SIMD instructions to speed them up. With 256-bit AVX2 instructions, for example, we can process L=8 lanes of 32-bit values at a time. A first approach could be to use this to compute the hash of L consecutive minimizer values at

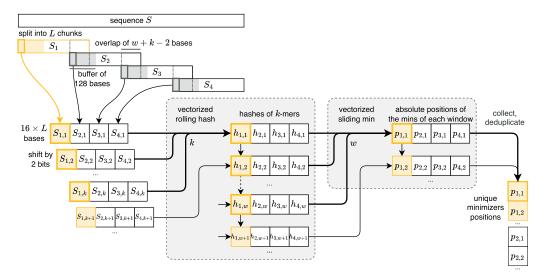


Figure 3 High-level view of the vectorized computation of minimizers using the building blocks presented in Section 3. The sequence is first split into L chunks (in this example L=4) that are processed in parallel. As described in Section 4.1, we load 128 bases at a time for each chunk, from which we extract 16 bases $S_{i,j}$ which are gathered in a SIMD register (one lane for each chunk, 32 bits per lane). This SIMD register is used to iterate over each lane by shifting and masking 2 bits at a time, and is passed as input to a vectorized rolling hash function that computes a hash $h_{i,j}$ for the k-mers in each lane. The absolute position $p_{i,j}$ of the minimum is then computed over a sliding window of w vectors of hashes for each lane. The positions are finally reordered to match the order of the original sequence and deduplicated to keep a unique occurrence of each position. At every step of the computation, the lane corresponding to the first chunk is highlighted in yellow. A bold outline indicates the bases in the first k-mer and the hashes corresponding to the first window.

the same time, and then to compute the minimizer of the L new windows all at once. Unfortunately, this is tricky due to the sequential nature of rolling hashing and sliding window minima.

Chunks. Instead of processing consecutive k-mers in parallel, we choose to split the input sequence into L equally long chunks that we process in parallel. This way, we compute one hash of each chunk in parallel, and then compute one minimizer position of a window of each chunk in parallel as well. This works, because the computations on the chunks are completely independent of each other. We let adjacent chunks overlap by $\ell-1$ characters, so that each window is fully contained in exactly one chunk. The total number of windows may not be divisible by L. In that case, we round up the chunk length and return the number of elements that was added as padding, so that these can be removed later. For simplicity, we omit that case from the code snippets.

4.1 Gathering the input

The parts that need special attention for SIMD instructions are the parallel reading of the input sequence, and the parallel deduplicating of the output minimizer positions (Section 4.2).

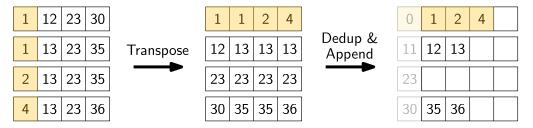


Figure 4 The SIMD version of the sliding window minimum algorithm produces minimizer positions for L chunks at a time. For example, the top row on the left indicates a SIMD vector containing the minimizer position of a window in each of the L=4 chunks. In the end, we want to return a single "flat" vector. Thus, we have to "deinterleave" the L lanes. First, we collect L minimizer positions of every chunk (the matrix on the left). Then, we transpose this matrix, so that the resulting SIMD vectors each correspond to a single chunk, with values corresponding to the first chunk highlighted. These are then compared with their preceding element (including the previous minimizer position, as shown in grey), and distinct elements are shuffled to the front. These positions are accumulated in a separate buffer for each chunk, and these buffers are finally concatenated into a single flat vector.

Algorithm 6 Pseudocode for splitting the input sequence into L chunks and iterating each chunk in parallel. The input sequence is assumed to be bitpacked, and indexed by bytes. Yields an iterator over L lanes of u32 values.

```
1: function GATHER(k, seq)
                                                                                   \triangleright Chunks overlap by k-1 bases.
 2:
         n = |\mathsf{seq}| - (k - 1)
                                                         \triangleright Number of k-mers. The sequence length is in bases.
 3:
         b = \lceil n/(4L) \rceil
                                                                                        > Byte offset between chunks.
 4:
         B = [(4b + (k-1))/16]
                                                                            ▷ Number of u32 covering each chunk.
                                                    {\bf \vartriangleright } \ L \times L \ matrix \ of \ {\tt u32}, \ represented \ as \ L \ {\tt u32xL} \ registers.
 5:
         M \leftarrow []
 6:
         for i in \{0, ..., B-1\} do
             if (i \mod L) = 0 then
 7:
                  for j in \{0, ..., L-1\} do

ightharpoonup Read a full SIMD-vector of data from each chunk.
 8:
 9:
                   M[j] \leftarrow \operatorname{seq}[jb+4i:jb+4i+4L] 
                                                                                            ▷ seq is indexed by bytes.

ightharpoonup Transpose the L \times L matrix.
10:
                  M \leftarrow \operatorname{transpose}(M)
             yield M[i \mod L]
                                                                                             ▷ Yield one u32 per lane.
11:
```

The first step is to read the actual bases. Since memory access instructions are relatively slow² compared to the SIMD operations we do on them, we cannot afford to read from each chunk one base or one byte at a time. We could read 32 bits at a time from each chunk using SIMD gather instructions, but these are relatively slow. Instead³, we read a full SIMD register of 256 bits worth of data at a time from each chunk, as shown in Algorithm 6. Then, we $transpose^{45}$ this matrix so that we obtain L SIMD registers, where the first contains the upcoming u32 for each chunk, the second contains the next u32 for each lane, and so on. Then, we use this transposed matrix as a buffer, and use it for the next 128 bases. After each 16 bases, we shift to the next u32x8 from it.

² https://uops.info/html-instr/VPGATHERDD YMM VSIB YMM YMM.html

³ As suggested by one of the anonymous reviewers, for which we thank them.

 $^{^4 \ \}mathtt{https://stackoverflow.com/questions/25622745/transpose-an-8x8-float-using-avx-avx2}$

⁵ https://github.com/rust-seq/simd-minimizers/blob/master/simd-minimizers/src/intrinsics/transpose.rs

4.2 Collecting and Deduplicating Positions

Transpose. The last step of the minimizer algorithm is to deduplicate the results, since adjacent windows often share the same minimizer position. In practice, deduplicating works best when the data to be duplicated is linear in memory. But the output of the vectorized sliding-window-minimum gives a u32x8 containing the position of one minimizer of each chunk. Thus, every L iterations we reuse the matrix transpose to obtain a u32x8 for each chunk, containing L consecutive minimizer positions⁶.

Dedup. We deduplicate each lane using the technique of [18]. This compares each element to the previous one, and compares the first element to the last minimizer of the window before. The distinct elements are then *shuffled* to the front of the SIMD vector using a lookup table and appended to a buffer for each lane (Figure 4). We end by concatenating all the per-lane buffers into a single vector of minimizer positions, and make sure to avoid duplicates between the end and start of adjacent lanes.

Super-k-**mers.** The deduplication can be amended to also find super-k-mers, which are sequences of consecutive windows sharing the same minimizer position. After comparing adjacent minimizer positions, we obtain a mask that determines the shuffle instruction to apply. Normally we shuffle the 32-bit minimizer positions directly. Instead, we can mask out the upper 16 bits and store there the index of its window. If we then shuffle those values, we obtain for each minimizer its position in the input text, and the position of the first window where this k-mer became a minimizer. This information is sufficient to recover all super-k-mers, and sequences longer than 2^{16} bp can be either split-up, or else it is easy to detect manually when the values wrapped.

5 Experimental Evaluation

Our code is available in the simd-minimizers crate that can be found at https://github.com/rust-seq/simd-minimizers. Part of the code was extracted into a separate library, packed-seq, for easy reuse in other projects, which is available at https://github.com/rust-seq/packed-seq. Both libraries support both AVX2 and NEON instruction sets, and we will now look at their performance. The code to reproduce the experiments is available in the simd-minimizers-bench subdirectory.

The experiments were run on an Intel Core i7-10750H with 6 cores with AVX2 running at a fixed frequency of 2.6GHz with hyperthreading disabled and cache sizes of 32 KiB (L1), 256 KiB (L2), and 12 MiB shared L3. Code is compiled with rustc 1.88.0-nightly⁷. As input, we use a fixed random string of 10⁸ bases, depending on the method encoded either as ASCII or as packed representation. Reported timings are the median of five runs and shown in nanoseconds per base.

In our experiments, we use parameter values for w and k as used by Kraken2 [33] (5,31), SSHash [29] (11,21), and Minimap2 [19] (19,19).

Tables 3 and 4 in Section A show equivalent results for the NEON architecture.

 $^{^{6} \ \}mathtt{https://github.com/rust-seq/simd-minimizers/blob/master/simd-minimizers/src/collect.rs}$

We have made sure that all layers of iterators are inlined into single function. This is usually needed for optimal performance. Even then, small changes to the generated code (by changing the source, or compiler version) can impact performance by as much as 20%.

taken when incrementally including more steps of the implementation, for (w, k) = (11, 21).

Part	ns/bp
Iterate the bases	0.15
+ collect to vector	0.30
+ iterate the delayed bases	0.30
+ ntHash	0.32
+ sliding window min	0.90
+ collect	1.48
+ dedup	1.61
+ canonical nthash	1.04
+ canonical strand	1.53
+ collect	2.03
+ dedup	2.20

■ Table 1 Total time per base ■ Table 2 Comparison of our simd-minimizers implementation against minimizer-iter [22] and a rescan implementation based on [20]. Times in ns/bp are shown for both forward and canonical minimizers (where supported), and for various (w, k) tuples. For our library, we test both ntHash and mulHash with multiple encodings of the input DNA: 1) 2-bit packed, 2) ASCII-ACGT that is packed onthe-fly, and 3) plain ASCII (for mulHash only).

(w,k):	(5, 31)		(11, 21)		(19, 19)	
Method	fwd.	cano.	fwd.	cano.	fwd.	cano.
minimizer-iter	25.30	32.84	26.96	33.93	26.81	34.04
Rescan ntHash	11.65	_	7.41	_	5.61	_
simd-minimizers ntHash						
 packed input 	1.69	2.28	1.61	2.20	1.64	2.16
- on-the-fly packing	1.92	2.50	1.84	2.42	1.91	2.42
Rescan mulHash	11.37	_	6.79	_	5.76	_
simd-minimizers mulHash						
 packed input 	1.85	2.49	1.74	2.40	1.78	2.42
- on-the-fly packing	g 2.12	2.70	2.05	2.62	2.05	2.65
- ASCII input	2.11	2.71	2.06	2.63	2.01	2.66

Incremental time usage. Table 1 shows the time usage for various incremental subsets of our method. To start, iterating the 8 chunks of the input and summing all bases takes 0.15 ns/bp. Appending all u32x8 SIMD vectors containing the bases to a vector takes 0.30 ns/bp, indicating that writing to memory induces some overhead. Collecting the second k-1-delayed stream of characters that leave the k-mer (by adding them to the non-delayed stream) has no additional overhead. Computing ntHash only takes 0.02ns/bp extra. The sliding window computation nearly triples the total time. Collecting the minimizer positions to a linear vector (i.e., transposing matrices and writing output for each of the 8 chunks) again incurs 50% overhead, ad deduplicating them actually again adds some time.

Going back a step, using canonical ntHash instead of forward ntHash takes 0.14 ns/bp extra, and determining the canonical strand (via a third ℓ -delayed stream and counting GT bases) takes another 0.49 ns/bp. As before, collecting and deduplicating are slow and add around 0.70ns/bp.

In conclusion, we see that iterating the chunks of the input and determining minimizers is quite fast, but that a lot of time must then be spent to "deinterleave" the output into a linear stream. As can be expected, canonical minimizers are slower to compute than forward minimizers, but the overhead is less than 50%, which seems quite low given that the ntHash and sliding window minimum computation are duplicated and a canonical-strand computation is added.

Full comparison. We compare against the minimizer-iter crate (v1.2.1) [22], which implements a queue-based sliding window minimum using wyhash [34] and also supports canonical minimizers. For an additional comparison, we optimized an implementation of the remap method with ntHash based on a code snippet by Daniel Liu [20].

Results are in Table 2 and Figure 5. minimizer-iter takes around 26ns/bp for forward and 33ns/bp for canonical minimizers, and its runtime does not depend much on w and k, because the popping from the queue is unpredictable regardless of w. Rescan starts out at

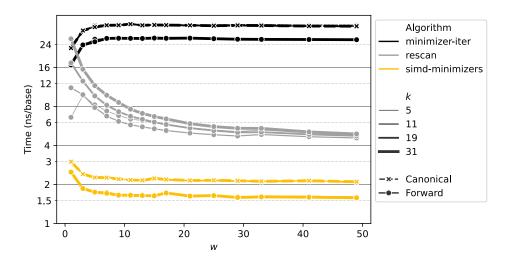


Figure 5 Running time (logarithmic) of minimizer-iter, rescan, and simd-minimizers for different values of w and k. For minimizer-iter and simd-minimizers, runtime is nearly independent of k, while both are slower when computing canonical minimizers (indicated with crosses).

11.7 ns/bp for w=5 and gets significantly faster as w increases, converging to around 5 ns/bp for $w\gg 100$. This is explained by the fact that rescan has a branch miss every time the current minimizer falls out of the window, which happens for roughly half the minimizers at a rate of 1/(w+1). Thus, as w increases, the method becomes more predictable and branch misses go down. Our method, simd-minimizers, runs around 1.61 ns/bp for forward and 2.20 ns/bp for canonical minimizers when given packed input, and therefore is $3.4\times$ to $6.8\times$ faster than the rescan method.

In Figure 5, we see that simd-minimizers' performance is mostly independent of k and w since it is mostly data-independent. Only for small $w \leq 5$ it is slightly slower due to the larger number of minimizers and hence larger size of the output.

As we use SIMD with 8 lanes, we could in theory expect up to $8\times$ speedup. In practice this is hard to reach because of constant overhead and because the overhead to work well with SIMD in the first place. In particular for large w, rescan benefits from very predictable and simple code and only outputs unique minimizer positions, making it very efficient. In SIMD, on the other hand, we use a data-independent algorithm, and output the minimizer position for every single window, which then has to be deduplicated. Thus, it is nice to see that even for large w, our method is over $3\times$ faster, despite this overhead.

ASCII input and mulHash. Apart from taking bit-packed input, simd-minimizers also works on ASCII-encoded DNA sequences of ACTG characters directly, which are then packed into values $\{0, 1, 2, 3\}$ for ntHash (in that order) during iteration. This is around 0.25 ns/bp slower, mostly because of the larger size of the unpacked input.

The mulHash variant is around 0.20ns/bp slower again, but works for any ASCII input. Performance on 100 MB of the Pizza&Chili corpus [5] English⁸ and Sources⁹ datasets is nearly identical to performance on the random DNA shown in Table 2.

⁸ https://pizzachili.dcc.uchile.cl/texts/nlang/

⁹ https://pizzachili.dcc.uchile.cl/texts/code/

Human genome. We also run simd-minimizers on the chromosomes of a human genome (T2T-CHM13v2.0¹⁰ [26]), of total size 3.2 Gbp. Here, computing forward minimizers takes 5.19 seconds, and canonical minimizers takes 6.71 seconds for (w, k) = (11, 21), which corresponds 1.67 and 2.15 ns/bp, which is within a few percent of Table 2.

Density. For (w,k)=(11,21), we get a density of 0.173 for forward minimizers, and 0.167 for canonical minimizers, which are both close to the expected density of $2/(w+1)=1/6\approx 0.167$. Changing to (w,k)=(19,19), we get density 0.098 for forward minimizers and 0.100 for canonical minimizers, both close to the expected density of 2/(19+1)=1/10=0.1. Thus, we see that in practice, ntHash is a sufficiently random hash function to approach the expected density of random minimizers with a perfectly uniform random hash function.

Multithreading. We test throughput in a multithreaded setting, by using 6 threads to process the 25 chromosomes (22, X, Y, and mitochondrion) in parallel. This way, processing takes 0.97 s (forward) and 1.27 s (canonical) when the input data is already loaded into memory, showing slightly above $5\times$ speedup. This is just below $6\times$ speedup as the chromosomes don't perfectly partition the data into 6 equal parts, so that some threads finish before others.

For applications, we recommend to simply call our library in parallel from multiple threads as needed.

6 Conclusions and Future Work

Our library simd-minimizers computes minimizer positions $3.4 \times$ to $6.8 \times$ faster than other methods. Using the library, only a single function call is needed to obtain the list of (canonical) minimizer positions, taking as input the (packed) DNA sequence and the parameters k and w. General ASCII input are also supported, allowing use cases such as sketching protein sequences. We hope that the community will adopt simd-minimizers as the standard library to compute random minimizers.

Future work. We chose to use the data-independent two-stacks method as the core of our algorithm, that returns the minimizer position of every window and then requires deduplication. Given the promising performance of rescan for large w in Table 2, an interesting alternative could be to go the opposite way and speed up the rescan step. This is particularly relevant when minimizers are sparse, as the number of branches may be small enough for linear scans to benefit from SIMD. Accelerating linear scans could also be useful for smaller inputs such as short reads, where splitting into 8 chunks may not be very efficient.

Another approach would be to use 512-bit AVX512 instructions and process 16 lanes in parallel. In theory that could be another $2\times$ faster, but in practice the collecting and deduplicating of values may become an even larger bottleneck. In simple experiments, without further profiling and optimizing the code, it is in fact 20% slower.

Additionally, implementing low-density schemes like the open-closed mod-minimizer [7, 11] would be a valuable extension.

 $^{^{10}\,\}mathrm{Available}$ at https://github.com/marbl/CHM13

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Results for NEON Architecture

The NEON experiments are run on a performance core on an Apple M1 chip with 4 efficiency and 4 performance cores. The performance core runs at 3.2GHz and has 128KiB of L1 cache, 12MiB of shared L2 cache (for the performance cores) and 8MiB of shared L3 cache (for the whole system).

adding steps of the implementation, for ture.

Part	$\rm ns/bp$
Gather and sum all bases	0.10
+ collect to vector	0.18
+ collect the delayed bases	0.19
+ ntHash	0.33
+ sliding window min	0.82
+ collect	0.95
+ dedup	1.38
+ canonical nthash	0.85
+ canonical strand	1.16
+ collect	1.37
+ dedup	1.82

■ Table 3 Time per base taken when ■ Table 4 Comparison of our simd-minimizers implementation against minimizer-iter [22] and a rescan im-(w,k) = (11,21) on NEON architec-plementation based on [20]. Times in ns/bp are shown for both forward and canonical minimizers (where supported), and for various (w, k) tuples, on NEON architecture.

(w,k):	(5,	31)	(11.	, 21)	(19	19)	
Method	fwd.	cano.	fwd.	cano.	fwd.	cano.	
minimizer-iter	12.89	16.60	14.26	18.08	14.20	18.04	
Rescan ntHash	5.95	_	3.92	_	2.82	_	
simd-minimizers ntHash							
 packed input 	1.42	1.85	1.38	1.82	1.38	1.83	
- on-the-fly packing	g 1.66	2.07	1.58	1.99	1.59	2.02	
Rescan mulHash	6.94	-	3.93	-	3.42	-	
simd-minimizers mulHash							
 packed input 	1.97	4.34	1.91	4.27	1.93	4.26	
- on-the-fly packing	2.11	4.37	2.04	4.41	2.03	4.30	
- ASCII input	2.12	4.39	2.06	4.44	2.04	4.29	

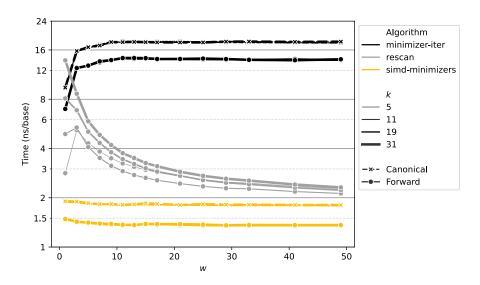


Figure 6 Running time of minimizer-iter, rescan and simd-minimizers on NEON architecture for different values of w and k.